

<Journal club Q&A>

1. Question: In the WGM portion vs t graph (figure (a) of the right column, page 4), why do the island excitation curve (fitting curve) converge to value less than 1?

- The time evolutions of WGM portion η in two distinct optical pathways can be obtained from corresponding governing equations (refer to the page 3 of the presentation)

i. Path 1: Analogy to three level system,

$$\eta_1 = \frac{n_m}{n_m + n_c + n_i}$$

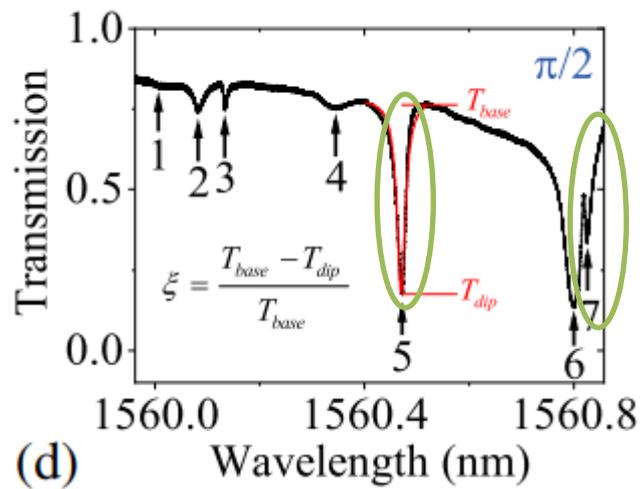
ii. Path 2: Analogy to two level system, (Eq. S4 in supplementary of Jiang et al., Science **358**, 344-347 (2017))

$$\eta_2 = \frac{n_m}{n_m + n_c} = \frac{2g_1}{\gamma_m - \gamma_c + 2g_1 + \sqrt{(\gamma_c - \gamma_m)^2 + 4g_1^2} \coth \left[\frac{1}{2} t \sqrt{(\gamma_c - \gamma_m)^2 + 4g_1^2} \right]}$$

- Looking at the path 2 case, η_2 does not converge to 1, but converge to $\frac{2g_1}{\gamma_m - \gamma_c + 2g_1 + \sqrt{(\gamma_c - \gamma_m)^2 + 4g_1^2}}$ after enough time t. Although they did not denote the solution of three level system case explicitly, it is easily expected that η_1 is hard to theoretically converge to the same value when the additional decay rates and coupling rates are arbitrary values.

2. Question: In the transmission vs wavelength graph (figure (b), (c), (d) of the first column, page 5), considering both pathways couple photons to arbitrary WGMs rather than specific one, why do other modes except for the mode 5 behave differently? (by the way are they WGM modes?)

- There are island modes, chaotic modes, and WGMs in highlighted Transmission spectra of page 5. The mode 5 and mode 7 are WGMs and the other modes are island or chaotic modes identified by relatively low Q factor. The other modes except for the mode 5 and 7 do not have to follow the rule they found. In addition, the coupling efficiency of the mode 7 is hardly discernible because of the Fano line shape due to the interference with low-Q chaotic modes. I think that is why they chose the mode 5 to study the coupling efficiencies of WGMs.



3. Question: How can they fabricate the single-axis symmetric cavity?

- Of course, when the single-axis symmetric cavity is fabricated using the photolithography and etching technique, the cavity has intrinsic fabrication error. However, they fitted the boundary with symmetry restriction, and they demonstrated that it can be by measuring unidirectional emission which indicates it has single-axis symmetry in previous works (Fig. S7 in supplementary of Jiang et al., Science **358**, 344-347 (2017) & C.L. Zou et al., IEEE J. Sel. Top. Quantum Electron **19**, 1 (2012))

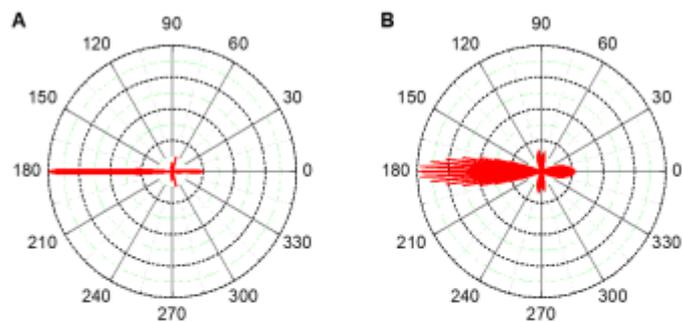


Fig. S7

Far-field emission patterns for a TM fundamental mode with $kR_0 \sim 72$ from ray (A) and wave (B) simulations, respectively.