Cavity-Modified Collective Rayleigh Scattering of Two Atoms

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Jinuk Kim
Quantum-Field Laser Laboratory
Department of Physics and Astronomy
Seoul National University, Korea

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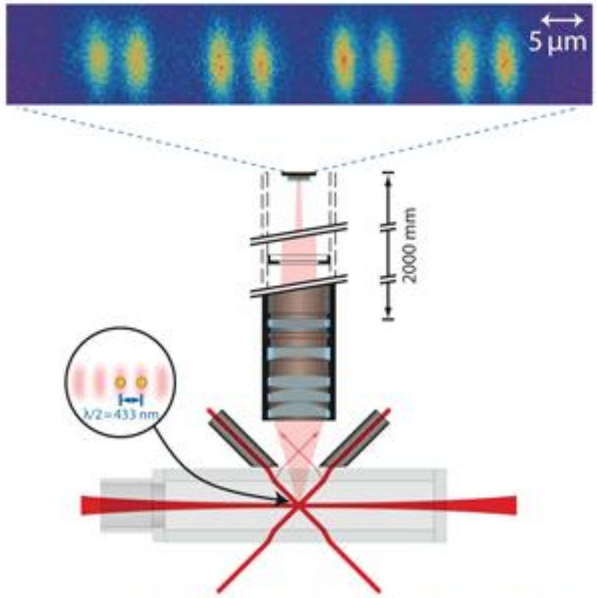


Fig. 1: The vacuum cell with lattice and imaging system: Individual cesium atoms can be trapped and observed.

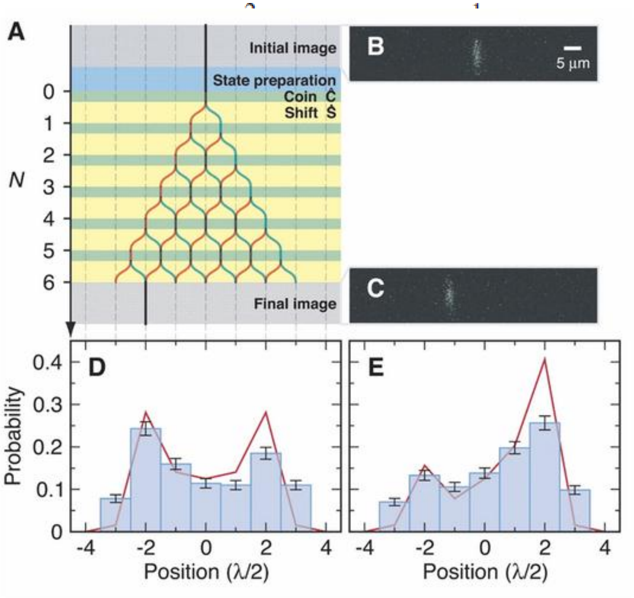
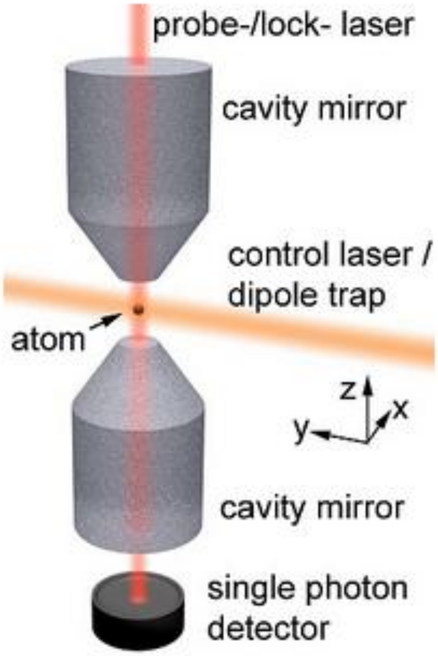
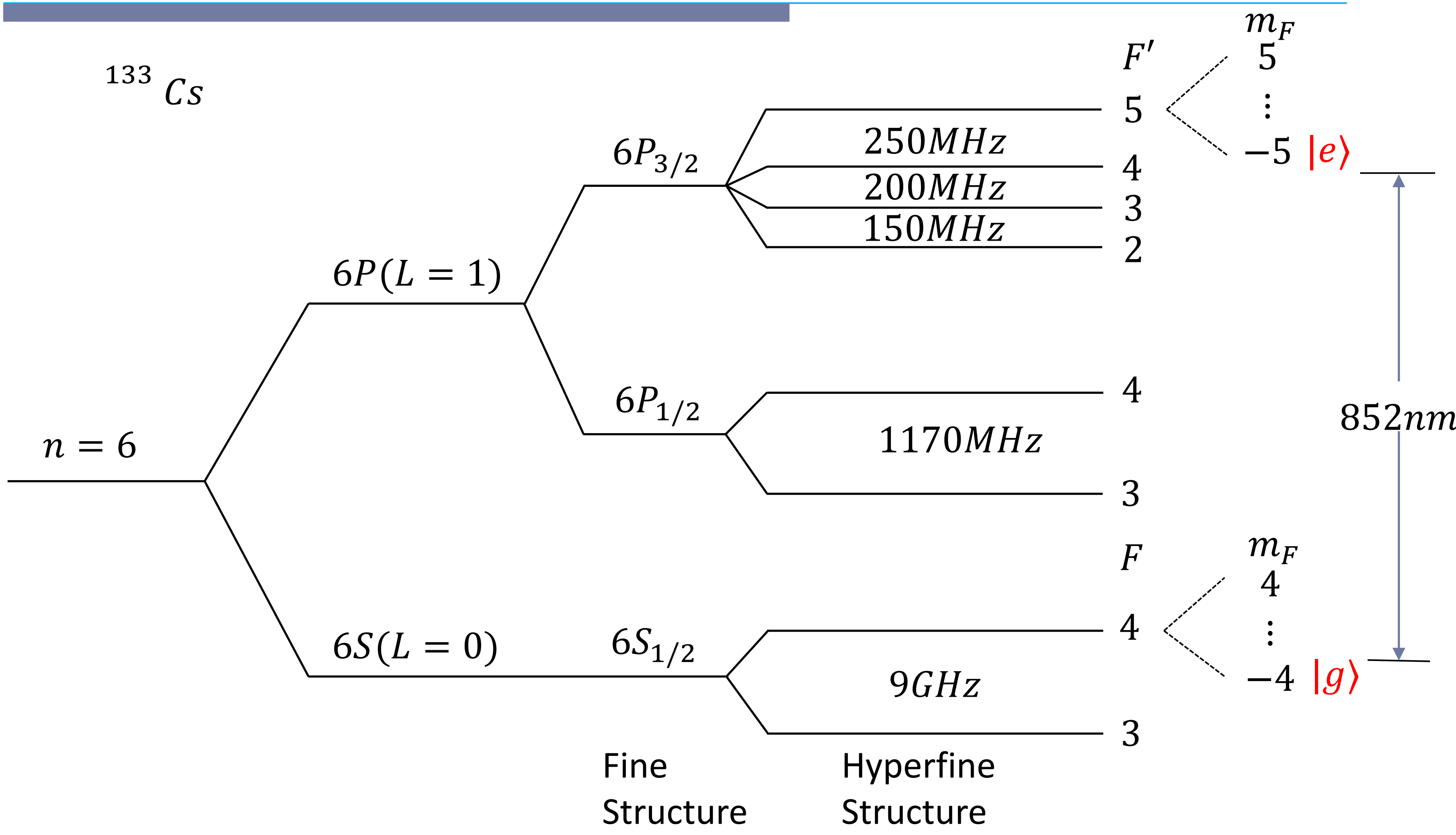


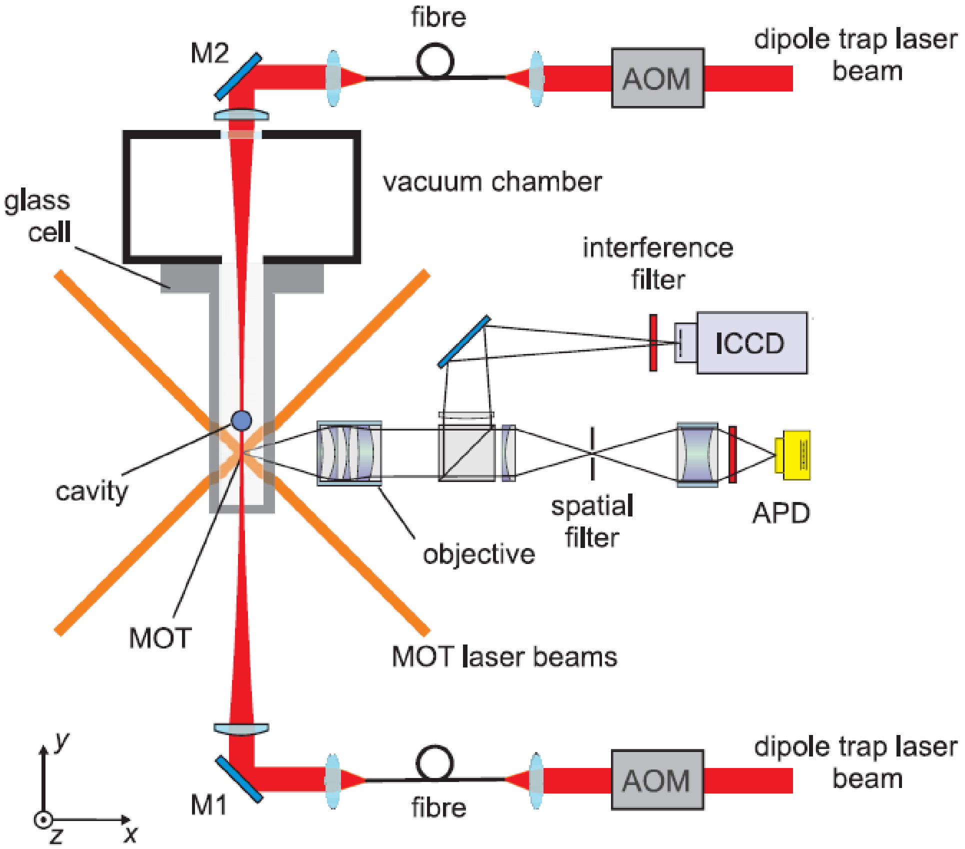
Fig. 3: Splitting an atom into many components and letting them interfere causes the double-peaked probability distribution. In every shot, the atom moves a small number of lattice sites.



Cesium

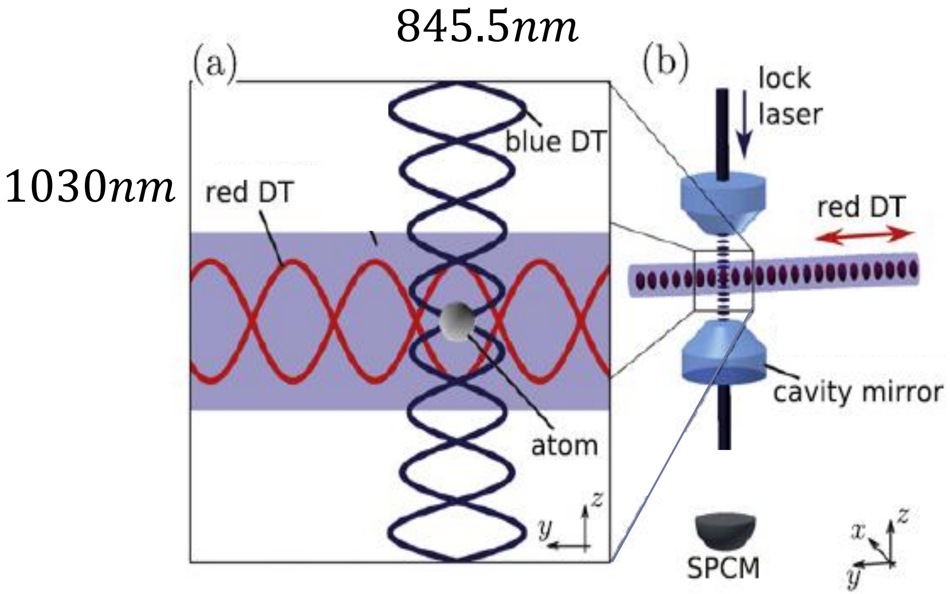

^{133}Cs
$n=6$
$6P(L=1)$
$6S(L=0)$
$6P_{3/2}$
$6P_{1/2}$
$6S_{1/2}$
$250MHz$
$200MHz$
$150MHz$
$1170MHz$
$9GHz$
F'
5
4
3
2
4
3
F
4
3
m_F
5
⋮
-5 $|e\rangle$
m_F
4
⋮
-4 $|g\rangle$
$852nm$
Fine Structure
Hyperfine Structure

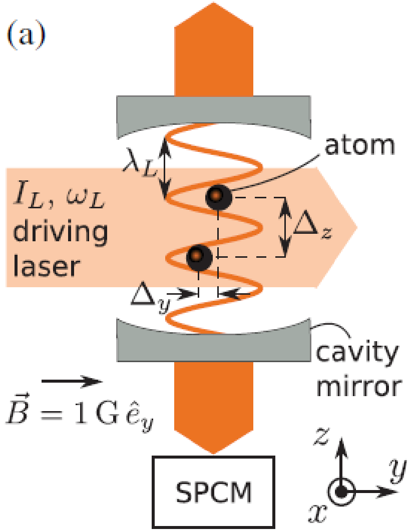
Setup



$1030nm$

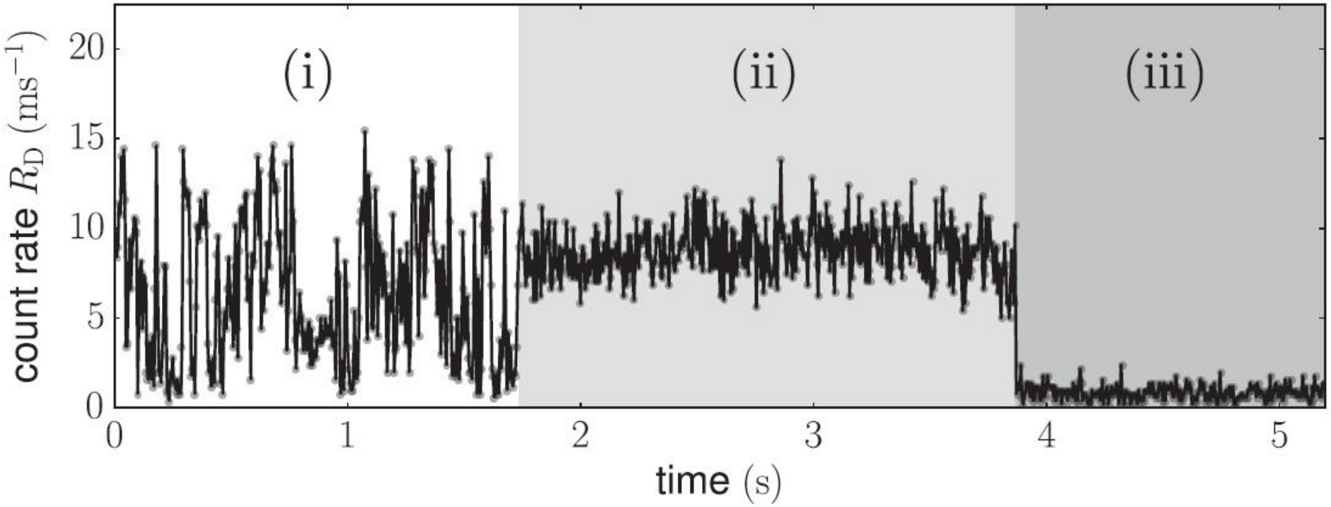
Setup


845.5nm
1030nm
(a)
blue DT
red DT
atom
y
z
(b)
lock laser
red DT
cavity mirror
SPCM
x
y
z

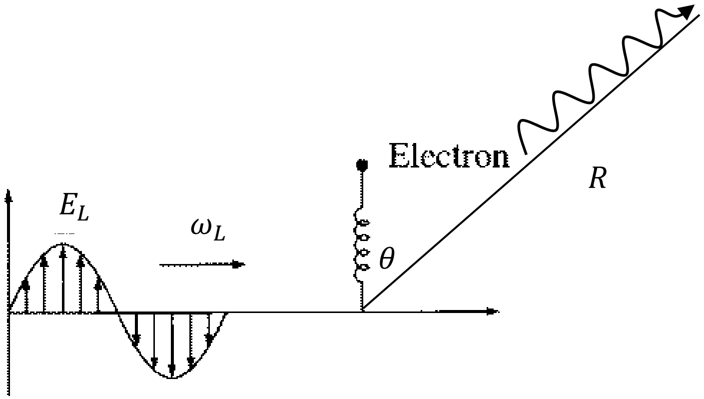

(a)
λ_L
atom
I_L, ω_L driving laser
Δ_z
Δ_y
cavity mirror
$\vec{B} = 1\,\mathrm{G}\,\hat{e}_y$
SPCM
z
y
x

Result

two atoms one atom no atom



Oscillator model



$$\ddot{x} + \Gamma_\omega \dot{x} + \omega_0^2 x = -eE_L(t)/m_e$$

$$p = \alpha E_L$$

$$\Delta \equiv \omega_L - \omega_0 \ll \omega_0$$

$$\alpha = 6\pi\epsilon_0 c^3 \frac{\Gamma/\omega_0^2}{\omega_0^2 - \omega_L^2 - i(\omega_L^3/\omega_0^2)\Gamma} \approx \frac{6\pi\epsilon_0 c^3}{\omega_L^2} L[\Delta]$$

$$where \;\; L[\Delta] = (-2\Delta\Gamma + i\Gamma^2)/(\Gamma^2 + 4\Delta^2)$$

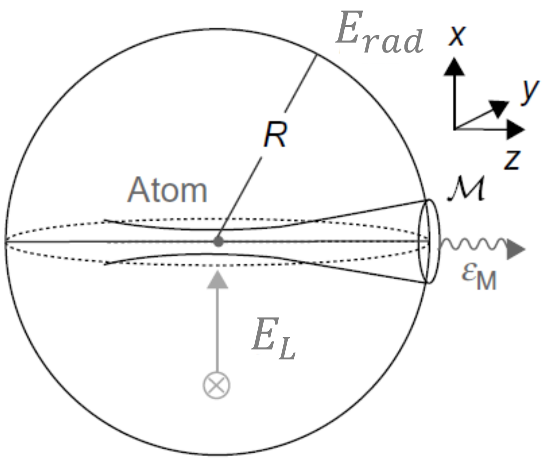
p : dipole moment

α : polarizability

$L[\Delta]$: atomic line function

$$\boldsymbol{E_{rad}(R,\theta)} \underset{\underset{R \gg \lambda}{\uparrow}}{\boldsymbol{\approx}} \boldsymbol{\frac{k^2 sin\theta}{4\pi\epsilon_0}\frac{e^{ikR}}{R}\alpha E_L} \underset{\underset{sin\theta \approx 1}{\uparrow}}{\boldsymbol{\approx}} \boldsymbol{\frac{k^2}{4\pi\epsilon_0}\frac{e^{ikz+\frac{ik\rho^2}{2z}}}{z}\alpha E_L}$$

Scattering into cavity mode



$$E_M(\rho,z) = e_M(\rho,z)\varepsilon_M/\sqrt{\epsilon_0 c}$$

ε_M : mode amplitude

Gaussian beam

$$e_M(\rho,z) = \left(\frac{2}{\pi\tilde{w}^2}\right)^{\frac{1}{2}} \exp\left(-\frac{\rho^2}{\tilde{w}^2} + ikz + ik\frac{\rho^2}{2R(z)} - i\zeta(z)\right)$$

$$\approx \left(\frac{2}{\pi\tilde{w}^2}\right)^{\frac{1}{2}} \exp\left(-\frac{\rho^2}{\tilde{w}^2} + ikz + ik\frac{\rho^2}{2z} - i\frac{\pi}{2}\right)$$

↑

$z \gg z_R$

$$\tilde{w} = w\sqrt{1+(z/z_R)^2} \approx wz/z_R$$

$$\zeta(z) = \tan^{-1}\frac{z}{z_0}$$

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$

$$\varepsilon_M = \sqrt{\epsilon_0 c}\int e_M^* E_{rad} 2\pi\rho d\rho$$

$$\therefore\ \boldsymbol{E_M = i\beta E_L} \quad \textit{where}\ \beta = \frac{k}{\pi w^2}\frac{\alpha}{\epsilon_0}$$

Cavity field

① ⇝ $(1+r^2+r^4+\cdots)E_M e^{ikz}e^{-i\phi_z}$

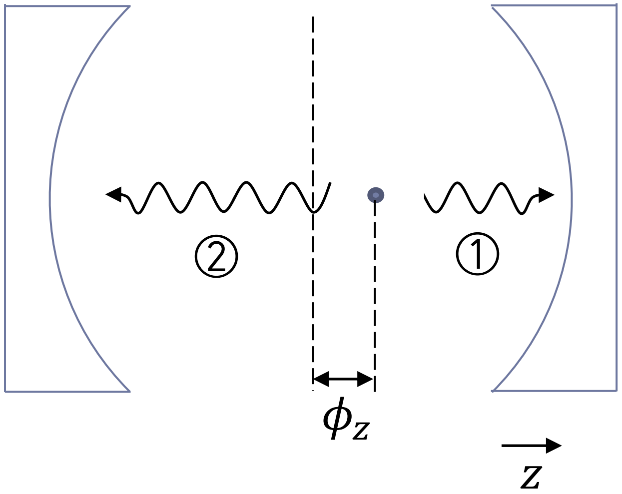
⇜ $(r+r^3+r^5+\cdots)E_M e^{-ikz}e^{i\phi_z}$

② ⇜ $(1+r^2+r^4+\cdots)E_M e^{-ikz}e^{-i\phi_z}$

⇝ $(r+r^3+r^5+\cdots)E_M e^{ikz}e^{i\phi_z}$

+) $r \approx 1$

$$E_C = \frac{2E_M}{1-r^2}\cos(\phi_z)$$



cavity field : $E_c\left(e^{ikz}+e^{-ikz}\right)$

Cavity field

$$E_M = i\beta E_L$$

$$\Rightarrow \quad E_M = i\beta(E_L e^{i\phi_y} + E_C e^{i\phi_z} + E_C e^{-i\phi_z})$$

$$\Rightarrow \quad E_M = i\beta\left(E_L e^{i\phi_y} + 2E_C \cos(\phi_z)\right)$$

$$E_C = 2E_M/(1-r^2)\cos(\phi_z)$$

$$\Rightarrow \quad (1-r^2)E_C/2 = i\beta\left(E_L e^{i\phi_y}\cos(\phi_z) + 2E_C \cos^2(\phi_z)\right)$$

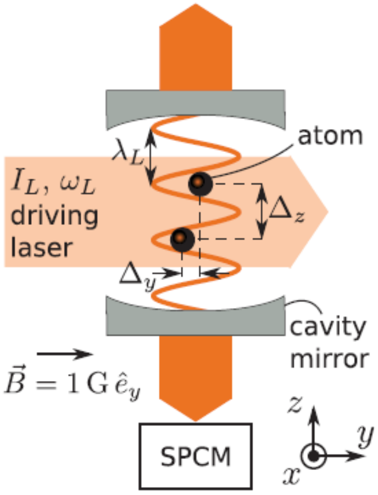
$$\Rightarrow \quad (1-r^2)E_C/2 = i\beta\left(E_L\left(1+e^{i\phi_y}\cos(\phi_z)\right) + 2E_C(1+\cos^2(\phi_z))\right)$$

$$G \equiv \frac{1}{N}\left(1+\exp[i\phi_y]\cos[\phi_z]\right)$$

$$H \equiv \frac{1}{N}\left(1+\cos^2[\phi_z]\right)$$

$$C \equiv \frac{g^2}{\kappa\Gamma}$$

$$g \equiv \sqrt{2\pi\Gamma c/(2k_L^2 V)}$$



$$\therefore\ \boldsymbol{E_C = -\frac{E_L}{2}\frac{NG}{\dfrac{i}{2CL[\Delta]} + NH}}$$

freespace limit	$\kappa \to \infty$	$\lvert E_c\rvert^2 \propto N^2$
perfect cavity limit	$\kappa \to 0$	$\lvert E_c\rvert^2$ is indep of N

Intracavity photon number